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Revisiting weight-restricted DEA models: A modified single-stage approach and an application to school efficiency evaluation

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ABSTRACT

Dynamic educational systems require continuous evaluation of performance and efficiency to respond effectively to structural, managerial, and social changes. Educational organizations can be assessed based on their objectives and missions, as well as how efficiently they use available resources. This study employs multiplier Data Envelopment Analysis (DEA) models to examine computational and conceptual issues that arise when very small lower bounds are imposed on input and output weights. It is shown that the conventional single-stage DEA approach, particularly under weight restrictions, may generate efficiency targets with negative input values, which are theoretically and practically invalid. To address this limitation, a modified single-stage formulation inspired by the two-stage framework of Ali and Seiford is proposed, allowing for the simultaneous and consistent evaluation of radial and non-radial inefficiencies while preventing infeasible efficiency targets. An empirical application to non-profit schools in Fars Province demonstrates that arbitrarily small weight bounds can lead to misleading efficiency projections, whereas the proposed model produces more realistic and interpretable results, highlighting the need to revise standard single-stage DEA formulations when weight restrictions are applied in educational performance assessment.

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1. Introduction

Educational systems, as one of the fundamental pillars of sustainable development, are continuously exposed to structural, economic, and social changes. Consequently, systematic performance and efficiency evaluation of such systems is indispensable. Any educational system or organization can be assessed based on its objectives, mission, and the extent to which it utilizes available resources and facilities efficiently. In this context, Data Envelopment Analysis (DEA) has emerged as a powerful non-parametric technique for evaluating the relative efficiency of decision-making units (DMUs) and has been widely applied in education, banking, industry, environmental studies, and sustainability analysis (Charnes et al., 1978; Banker et al., 1984).

In the field of education, DEA has been extensively used to evaluate the efficiency of schools, universities, and educational districts, allowing policymakers to identify best-performing institutions and sources of inefficiency (Bessent et al., 1982; Charnes et al., 1981; Ruggiero, 1996; Thanassoulis, 1996). These studies demonstrate that DEA is particularly suitable for educational evaluation due to the multi-input–multi-output nature of teaching and learning processes, where outcomes such as academic achievement, graduation rates, and quality indicators must be considered simultaneously.

DEA models, particularly in their multiplier and envelopment forms, enable the simultaneous evaluation of multiple heterogeneous inputs and outputs without requiring an explicit specification of a production function. However, one of the main challenges associated with multiplier DEA models is the excessive flexibility of weights, which may lead to unrealistic or extreme weight values and, consequently, reduce the discriminatory power of the model. This issue is especially critical in school efficiency studies, where unconstrained weights may ignore important educational inputs such as teacher experience or student background (Thanassoulis and Dunstan, 1994; Johnes, 2006). To address this problem, various forms of weight restrictions have been introduced and extensively studied in the DEA literature (Podinovski, 1999; 2004; 2007; Cook and Zhu, 2008; Allen et al., 1997; Thanassoulis et al., 2004).

In the conventional single-stage approach, multiplier DEA models impose a very small positive lower bound ε on all input and output weights. This lower bound is chosen arbitrarily. The aim is to evaluate radial efficiency as the main objective while also considering non-radial inefficiencies as a secondary objective. However, Ali and Seiford (1993) showed that using extremely small values of ε can cause serious computational problems and numerical inaccuracies because of the limited precision of computer-based optimization. To address this issue, they proposed a two-stage optimization approach. In this method, radial efficiency is evaluated in the first stage, and non-radial improvements are considered in the second stage.

Despite the advantages of the two-stage approach, its application to DEA models with weight restrictions introduces additional conceptual and practical challenges. Podinovski (2007) showed that applying the standard second-stage optimization directly to weight-restricted DEA models may result in efficiency targets with negative input values, which are generally meaningless from an economic and managerial perspective. Such issues are particularly problematic in educational settings, where negative values for resources such as teaching hours or expenditures are infeasible (Johnes and Yu, 2008). To resolve this issue, a modified second-stage optimization procedure was proposed that prevents the occurrence of such infeasible targets.

In recent years, DEA has been increasingly applied not only to static efficiency evaluation but also to dynamic productivity

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analysis, sustainability assessment, and sustainable development studies in education. For example, the Malmquist productivity index has been used to examine productivity changes in schools and universities over time (Rashidi et al., 2014; Worthington, 2001). Moreover, DEA-based frameworks integrating undesirable outputs and social indicators have been employed to evaluate the broader performance of educational institutions within sustainable development contexts (Hosseini et al., 2018; Rashidi et al., 2025).

Furthermore, hybrid approaches combining DEA with multi-criteria decision-making methods, such as the Balanced Scorecard and the Analytic Network Process, have enabled more comprehensive performance evaluations in educational systems (Fanati Rashidi, 2020; Kao and Hung, 2008). Several studies have also examined issues related to returns to scale, congestion, and weight control in school efficiency analysis, highlighting the importance of methodological rigor when DEA models are applied to real-world educational data (Ruggiero, 2004; Asil and Rashidi, 2015).

Nevertheless, the use of arbitrarily small lower bounds on weights in multiplier DEA models with weight restrictions may still lead to theoretically invalid results, even in the absence of numerical inaccuracies. This phenomenon can occur under both constant returns to scale (CRS) and variable returns to scale (VRS) assumptions (Charnes et al., 1978; Banker et al., 1984). This indicates that the problem is not merely computational but rather rooted in the theoretical formulation of the single-stage DEA model itself.

Accordingly, the main objective of this study is to investigate the potential shortcomings of the standard single-stage DEA approach in the presence of weight restrictions and to propose a theoretical modification of the single-stage formulation. The proposed modification leads to a new single-stage DEA model—presented in both multiplier and envelopment forms—that is theoretically sound for compactly representing two objectives: the evaluation of radial and non-radial inefficiency of a DMU. It is shown that the modified model avoids generating negative input targets and yields more meaningful efficiency projections.

Finally, the efficiency of non-profit schools in Fars Province is evaluated using weight-restricted DEA models. The results show that small lower bounds on weights may lead to the identification of efficient targets with negative inputs, whereas the modified model successfully overcomes this issue and provides more realistic efficiency assessments.

2. Production technology with weight restrictions

The bidirectional relationship between weight restrictions and production trade-offs provides a fundamental basis for configuring weight restrictions. In general, our objective is to reduce the computational errors caused by using extremely small lower bounds on input and output weights in the multiplier DEA model.

First, it is necessary to formulate the reciprocal relationship between weight restrictions and the production trade-off process, to examine the concept of production technology derived from DEA models with weight restrictions, and to demonstrate that the single-stage formulation of DEA models may lead to a misleading interpretation of efficiency targets.

Consider a decision-making unit (DMU) (X_o, Y_o), where $x \in R^m$ and $y \in R^s$, evaluated using the CRS multiplier DEA model with K weight restrictions. The vectors u and v denote the output and input weights, respectively, and the weight restrictions are defined as:

$$V^{\mathrm{T}}P_t - U^{\mathrm{T}}Q_t \geq 0, \; t = 1, 2, \ldots, K \tag{1}$$

The right-hand side of these equations is zero, representing homogeneous weight restrictions (Podinovski, 2004). The components of the vectors $P_t \in R^m$ and $Q_t \in R^s$ may be positive, negative, or zero.

Consider the output-oriented CRS multiplier DEA model incorporating weight restrictions. The radial efficiency of DMU_0 is the reciprocal of the optimal value of the following linear programming problem:

$$\begin{aligned} & \eta^* = \text{Min} \quad V^{\mathrm{T}}X_o \\ & U^{\mathrm{T}}Y_o = 1 \\ & V^{\mathrm{T}}X_j - U^{\mathrm{T}}Y_j \geq 0 \qquad j = 1, \ldots, n \\ & V^{\mathrm{T}}P_t - U^{\mathrm{T}}Q_t \geq 0 \qquad t = 1, \ldots, k \\ & \mathrm{U}, \mathrm{V} \geq 0 \end{aligned} \tag{2}$$

The dual form of this model is expressed as follows, where the dual constraints are equalities represented by non-negative vectors d and e:

$$\begin{aligned} & \eta^* = \text{Max} \quad \eta \\ & \sum_{j=1}^{n} \lambda_j X_j + \sum_{t=1}^{k} \pi_t P_t + d = X_o \\ & \sum_{j=1}^{n} \lambda_j Y_j + \sum_{t=1}^{k} \pi_t Q_t - e = \eta Y_o \\ & \pi, d, e, \lambda \geq 0, \eta \;\; free \end{aligned} \tag{3}$$

Clearly, the weight restrictions in the multiplier model (2) generate additional dual terms in model (3). Podinovski (2004) referred to these terms as production trade-offs. Model (3) projects DMU_0 onto the CRS efficiency frontier.

Definition 1. (Podinovski, 2004). The CRS technology with production trade-offs, denoted by T_CRS–TO, is defined as the set of all DMUs (x, y) $\in R^m \times R^s$ for which there exist intensity vectors $\lambda \in R^n$, $\pi \in R^k$, and non-negative slack vectors d $\in R^m$ and e $\in R^s$ such that:

$$\begin{aligned} & \textstyle\sum_{j=1}^{n} \lambda_j X_j + \sum_{t=1}^{k} \pi_t P_t + d = X_o \\ & \textstyle\sum_{j=1}^{n} \lambda_j Y_j + \sum_{t=1}^{k} \pi_t Q_t - e = Y_o \end{aligned} \tag{4}$$

Equation (4) describes an arbitrary DMU in the CRS technology. The terms involving π_t represent adjustments to this DMU through production trade-offs (P_t, Q_t). Accordingly, the DMU is modified by increasing inputs by d and reducing outputs by e. If all inputs and outputs remain non-negative, the DMU is considered a member of the technology T_CRS–TO.

The VRS technology is defined similarly by adding the constraint $\sum_{j=1}^{n} \lambda_j = 1$.

If weight restrictions are inconsistent, feasibility of models (2) and (3) cannot be guaranteed. Podinovski et al. (2013, 2015) proposed analytical and computational methods for examining the consistency of weight restrictions. In this study, weight restrictions are assumed to be consistent.

3. Modified single-stage model

Based on Definition 1, we propose a simple modification to the single-stage development of multiplier DEA models to overcome the issues highlighted earlier. The modified model identifies inefficient units on the frontier of the technologies T_CRS–TO and T_VRS–TO, ensuring non-negativity of target inputs and outputs.

The output-oriented modified single-stage model for identifying an efficient target within T_CRS–TO is formulated as:

$$\begin{aligned} & \eta^* = \text{Max} \quad \textstyle\sum_{i=1}^{m} \xi_i + \sum_{r=1}^{s} \xi_r \\ & \textstyle\sum_{j=1}^{n} \lambda_j X_j + \sum_{t=1}^{k} \pi_t P_t + d = X_o - \xi_i \\ & \textstyle\sum_{j=1}^{n} \lambda_j Y_j + \sum_{t=1}^{k} \pi_t Q_t - e = \eta Y_o + \xi_r \\ & \pi, d, e, \lambda \geq 0, \eta \;\; free \end{aligned} \tag{5}$$

The full formulation is:

X_o- $\xi_i \geq 0$
$\eta Y_o + \xi_r \geq 0$ (6)

Let (λ*, π*, d*, ξ*, ς*, η*) be an optimal solution to model (6). The efficient target DMU is defined as:

$$(X^*, Y^*) = (X_0 - \xi^*, \eta^* Y_0 + \varsigma^*) \quad (7)$$

It follows that (X*, Y*) satisfies the defining constraints with e = 0 and therefore belongs to the technology T_CRS–TO. Hence, the following result holds.

Theorem 1. The DMU (X*, Y*) is efficient in the technology T_CRS–TO.

By associating dual variables v, u, and w with constraints (6), the dual formulation of the modified single-stage model can be expressed as:

$$\text{Min} \quad (V^T + W^T)\, X_o \quad (8)$$
$$U^T Y_o = 1$$
$$V^T X_j - U^T Y_j \geq 0 \qquad j = 1, \dots, n$$
$$V^T P_t - U^T Q_t \geq 0 \qquad t = 1, \dots, k$$
$$v_i + w_i \geq \varepsilon \quad i = 1, \dots, m$$
$$u_r \geq \varepsilon \qquad r = 1, \dots, s$$
$$V, W \geq 0$$

4. Efficiency evaluation of non-profit schools using modified models

In this section, a set of activities and characteristics of educational units are considered as input and output indicators. The models introduced in the previous section are employed to evaluate these units.

Statistical data were collected and are presented in Table 1. These data are used as inputs and outputs and analyzed using the previously discussed models. The results obtained from solving Models (2) and (3) are reported in Tables 2 and 3, respectively.

Table 1
Input and output data of schools.

DMU	Teaching Experience (X1)	Academic Degree Level (X2)	Cost (Million Tomans) (X3)	Primary GPA (X4)	Middle School GPA (Y1)	Admission Rate (Y2)
A	18	1.29	242.22	18.55	19.66	0.92
B	18	1.35	223.46	17.97	19.14	0.78
C	21	1.50	196.95	19.86	18.03	0.98
D	21	1.25	188.25	17.50	16.30	0.77
E	18	1.35	179.59	18.80	16.34	1.00
F	14	1.63	110.92	17.90	16.38	0.85
G	20	1.50	101.90	18.44	18.82	1.00
H	20	1.40	95.74	19.28	17.58	0.99
I	23	1.20	94.79	17.34	17.52	0.79
J	17	1.21	85.10	18.50	16.67	0.97
K	19	1.46	84.96	17.30	17.70	0.75
L	20	0.998	84.02	18.30	17.53	0.84
M	19	1.36	83.53	17.56	17.44	0.83
N	14	1.16	79.41	17.79	16.24	0.75
O	14	1.37	77.84	18.76	18.17	1.00
P	16	1.58	75.08	18.04	15.79	0.81
Q	12	1.33	73.16	19.00	19.00	1.00
R	19	1.28	68.15	19.54	18.55	0.99
S	17	1.47	64.48	17.20	18.08	0.84
T	18	1.30	63.51	19.25	17.23	0.88
U	16	1.67	63.33	19.75	16.23	1.00
V	17	1.17	59.97	18.54	17.26	1.00
W	19	1.09	59.23	16.47	18.01	0.89
X	19	1.28	58.97	18.93	15.12	0.93
Y	16	1.25	58.42	17.58	14.58	0.77
Z	16	1.15	50.58	16.09	17.28	0.89

The production trade-offs are defined using two vectors, P and Q. Vector P is defined as (0, 1/2, -1, 1/6) and vector Q is defined as (1/8, 0).

Fig. 1 illustrates the efficiency scores of non-profit schools evaluated using the weight-restricted DEA Model (2), highlighting the relative performance of decision-making units under the proposed framework.

Based on the solutions of the above models, it can be observed that decision-making units (Z, W, V, Q, and L) are identified as convergently efficient by both Models (2). Moreover, DMUs (O and G) are also found to be efficient under both models. As observed, the models produce closely comparable results in identifying efficient units.

Table 2
Input and output weights and efficiency scores using model (2).

DMU	Efficiency	Output weight U1	Output weight U2	Input weight V1	Input weight V2	Input weight V3	Input weight V4
A	1.0007	0.0508	—	0.0091	0.0451	0.0140	—
B	1.0011	0.0522	—	0.0093	0.0463	0.0139	—
C	1.0972	0.0613	1.0204	0.0002	0.0539	—	—
D	1.1470	0.0072	—	0.0002	0.0670	0.6409	—
E	1.0153	0.0570	1.0101	0.0017	0.0523	—	—
F	1.1175	0.0564	1.0309	0.0040	0.0592	—	—
G	1.0000	0.0570	1.0000	0.0017	0.0523	—	—
H	1.0535	0.0573	1.0101	0.0002	0.0533	—	—
I	1.0822	0.0615	1.0861	0.0002	0.0624	—	—
J	1.0287	0.0076	1.0309	0.0017	0.0539	—	—
K	1.0591	—	1.0101	1.0101	0.0501	—	—
L	1.0000	—	1.0102	1.0102	0.0508	—	—
M	1.0881	—	—	0.0157	0.0067	—	—
N	1.0842	—	—	0.0037	0.0505	—	—
O	1.0000	—	—	0.0042	0.0620	—	—
P	1.1883	—	—	0.0516	—	—	—
Q	1.0000	—	—	0.0818	—	—	—
R	1.0616	0.0526	0.9179	0.0035	0.0495	0.0030	—
S	1.0121	0.0049	1.0000	0.0099	0.0490	0.0091	—
T	1.1517	0.0553	1.0000	0.0156	0.0396	0.0111	—
U	1.0168	0.0580	1.0752	0.0252	0.0576	0.0080	—
V	1.0000	—	1.0709	0.0252	—	0.0056	—
W	1.0000	0.0555	—	0.0059	—	0.0018	—
X	1.0850	0.0120	—	0.0059	—	0.0185	—
Y	1.2144	0.0578	—	0.0059	—	0.0131	—
Z	1.0000	—	—	0.0059	—	0.0203	—

4. Results

Identifying productivity targets for inefficient DMUs in CRS and VRS DEA models can be examined as a two-stage process. According to Ali and Seiford (1993), the first stage identifies the radial input- or output-oriented projection of the DMU under evaluation onto the technology frontier.

The objective of the second stage is to eliminate any remaining non-radial inefficiencies in the radial target DMU. This is achieved by maximizing the sum of input and output slacks.

Conventional single-stage multiplier DEA models impose a theoretically negligible lower bound (0 < ε) on input and output weights. Their dual envelopment models simultaneously evaluate radial input or output efficiency as the primary objective and consider input and output slacks as a secondary objective associated with observed activities.

Table 3
Values of d and e and efficiency scores of schools using model (3)

DMU	d1 (Teaching Experience)	d2 (Academic Degree Level)	d3 (Cost, Million Tomans)	d4 (Primary GPA)	e1 (Middle School GPA)	e2 (Admission Rate)
A		0.0374	173.85			0.0725
B		0.1407	157.50			0.1828
C			101.77		0.2875	
D	0.8117	0.0918	125.31			0.0416
E		0.0807	108.37	1.318		
F			42.78	0.7787		0.3909
G		0.0524	10.42			
H			32.43			
I	2.9963					
J		0.0385	24.75			
K		0.0308	22.27			
L			19.76			
M		0.1886	7.01			
N			7.17			
O						
P		0.3434				
Q						
R					0.451	
S		0.1128	1.24		0.3217	0.0716
T				0.1128		0.0011
U		3.1661		0.3217		
V						
W						
X	1.3635		2.073			
Y						0.073
Z						

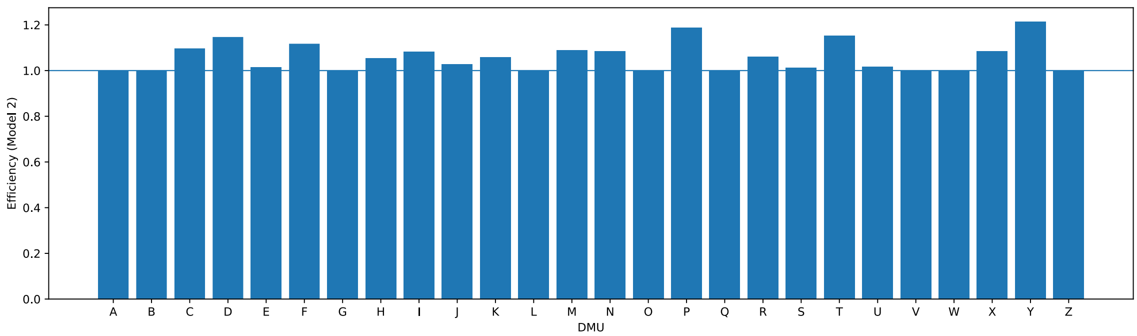

Fig. 1. Efficiency scores of non-profit schools obtained from the weight-restricted DEA Model (2).

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