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On the Rucklidge time-delayed chaotic system for nonlinear double convection: Adaptive control, synchronization and LabVIEW implementation

N. Bernardara *, A. Lafaurie, F. I. Saad

Faculté des Sciences, Université de Nice Sophia Antipolis, parc Valrose, 06108 Nice, France

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A B S T R A C T

There is great interest shown in the literature in the discovery of chaotic motion and oscillations in nonlinear dynamical systems arising in physics, chemistry, biology, and engineering. Chaotic systems have many important applications in science and engineering. This paper discusses the Rucklidge chaotic system for nonlinear double convection and time-delayed Rucklidge chaotic system. When the convection takes place in a fluid layer rotating uniformly about a vertical axis and in the limit of tall thin rolls, convection in an imposed vertical magnetic field and convection in a rotating fluid layer are both modeled by Rucklidge's three-dimensional system of ordinary differential equations, which produces chaotic solutions. This paper starts with a detailed description of the Rucklidge's nonlinear double convection system and the parameter values for which the Rucklidge system exhibits chaotic behavior. Next, an adaptive feedback controller is designed for the global chaos control of the time delayed Rucklidge chaotic system with unknown parameters. Furthermore, an adaptive feedback controller is designed for the global chaos synchronization of the identical Rucklidge chaotic system with its time-delayed Chaotic System. All the main results derived in this work are illustrated with MATLAB simulations. Finally, the circuit design of the novel A 3-D chaotic system is implemented in LABVIEW to validate the theoretical chaotic model.

1. Introduction

Chaos theory describes the qualitative study of unstable aperiodic behaviour in deterministic nonlinear dynamical systems. A dynamical system is called *chaotic* if it satisfies the three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions (Azar and Vaidyanathan, 2015). Chaos theory has applications in several areas in Science and Engineering.

A significant development in chaos theory occurred when Lorenz discovered a 3-D chaotic system of a weather model (Lorenz, 1963). Subsequently, Rössler found a 3-D chaotic system (Rössler, 1976), which is algebraically simpler than the Lorenz system. Indeed, Lorenz's system is a seven-term chaotic system with two quadratic nonlinearities, while Rössler's system is a seven-term chaotic system with just one quadratic nonlinearity.

Some well-known paradigms of 3-D chaotic systems are Arneodo system (Arneodo et al., 1981), Sprott systems (Sprott, 1994), Chen system (Chen and Ueta, 1999), Lü-Chen system (Lü and Chen, 2002), Liu system (Liu et al., 2004), Cai system (Cai and Tan, 2007), Tigan system (Tigan and Opriş, 2008), etc.

In the last two decades, many new chaotic systems have been also discovered like Li system (Li, 2008), Sundarapandian systems (Sundarapandian and Pehlivan, 2012; Sundarapandian, 2013), Vaidyanathan systems (Vaidyanathan and Madhavan, 2013; Vaidyanathan et al., 2015a; Vaidyanathan and Volos, 2015), Pehlivan systems (Sundarapandian and Pehlivan, 2012; Pehlivan et al., 2014), Pham systems (Pehlivan et al., 2014; Pham et al., 2015), Jafari system (Jafari and Sprott, 2013), etc.

Chaos theory has applications in several fields of science and engineering such as lasers (Behnia et al., 2013), oscillators (Tuwankotta, 2006), chemical reactions (Vaidyanathan, 2015a), biology (Kyriazis, 1991), ecology (Sahoo and Poria, 2014), artificial neural networks (Huang and Huang, 2008; Sun et al., 2010), robotics (Islam and Murase, 2005; Volos et al., 2013), electrical circuits (Matouk, 2011; Volos et al., 2015), cryptosystems (Volos et al., 2013), memristors (Pham et al., 2014; Volos et al., 2015), etc.

In fluid mechanics modelling, cases of two-dimensional convection in a horizontal lawyer of Boussinesq fluid with lateral constraints were studied by Rucklidge (Rucklidge, 1992). When the convection takes place in a fluid layer rotating uniformly about a vertical axis and in the limit of tall thin rolls, convection in an imposed vertical magnetic field and convection in a rotating fluid layer are both modelled by a new three-dimensional system of ordinary differential equations, which produces chaotic solutions like the Lorenz system (Lorenz, 1963).

In this paper, we first discuss the Rucklidge chaotic system (Rucklidge, 1992) for nonlinear double convection and detail its qualitative properties.

Next, we derive an adaptive control law that stabilizes the Rucklidge time delayed chaotic system with unknown system parameters. Furthermore, we also derive an adaptive control law that achieves global chaos synchronization of Rucklidge chaotic systems with is Rucklidge time delayed chaotic system. Our main adaptive control results for global chaos stabilization and synchronization are established using Lyapunov stability theory.

Synchronization of chaotic systems is a phenomenon that may occur when a chaotic oscillator drives another chaotic oscillator. Because of the butterfly effect which causes the

* *Corresponding Author.*
Email Address: nobernardara@yahoo.fr *(N. Bernardara)*
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exponential divergence of the trajectories of two identical chaotic systems started with nearly the same initial conditions, synchronizing two chaotic systems is seemingly a very challenging problem.

In most of the synchronization approaches, the master-slave or drive-response formalism is used. If a particular chaotic system is called master or drive system, and another chaotic system is called slave or response system, then the idea of synchronization is to use the output of the master system to control the response of the slave system so that the slave system tracks the output of the master system asymptotically.

In the chaos literature, a variety of techniques have been proposed to solve the problem of chaos synchronization such as PC method (Pecora and Carroll, 1990), active control method (Vaidyanathan and Rasappan, 2010; Vaidyanathan et al., 2015a), adaptive control method (Vaidyanathan and Rajagopal, 2011d; Vaidyanathan et al., 2015b), backstepping control method (Rasappan and Vaidyanathan, 2012), fuzzy control method (Kuo, 2011; Chadli and Zelinka, 2014), sliding mode control method (Vaidyanathan and Sampath, 2011), etc.

This paper is organized as follows. In Section 2, we describe the 3-D Rucklidge chaotic system for nonlinear double convection. In Section 3, we describe the qualitative properties of the Rucklidge chaotic system. In Section 4, we detail the adaptive control design for the global chaos stabilization of the 3-D Rucklidge chaotic system with unknown parameters. In Section 5, we detail the adaptive control design for the global and exponential synchronization of the identical novel Rucklidge chaotic systems. In Section 6, we describe the LABVIEW implementation of the Rucklidge chaotic system and the control results for the Rucklidge chaotic system. In Section 7, we give a summary of the main results obtained in this research work.

2. Rucklidge chaotic system for double convection

In this section, we describe the Rucklidge chaotic system (Rucklidge, 1992) for nonlinear double convection. Rucklidge chaotic system is modelled by the 3-D nonlinear dynamics

$$\begin{cases} \dot{x}_1 = -ax_1 + bx_2 - x_2x_3 \\ \dot{x}_2 = x_1 \\ \dot{x}_3 = -x_3 + x_2^2 \end{cases} \quad (1)$$

where x_1, x_2, x_3 are state variables and a, b are constant, positive, parameters of the Rucklidge system.

The system (1) is *chaotic* when the parameter values are taken as

$$a = 2b = 6.7 \quad (2)$$

For numerical simulations, we take the initial conditions of the Rucklidge system (1) as:

$$x_1(0) = 1.2, x_2(0) = 0.8, x_3(0) = 1.4 \quad (3)$$

The Lyapunov exponents of the 3-D chaotic system (1) for the parameter values (2) and the initial conditions (3) are numerically calculated as

$$L_1 = 0.1877, L_2 = 0, L_3 = -3.1893 \quad (4)$$

We note that the sum of the Lyapunov exponents of the Rucklidge chaotic system (1) is negative. Thus, the Rucklidge chaotic system (1) is dissipative. The Kaplan-Yorke dimension of the Rucklidge chaotic system (1) is derived as

$$D_{KY} = 2 + \frac{L_1+L_2}{|L_3|} = 2.0589 \quad (5)$$

Fig. 1 shows the 3-D phase portrait of the Rucklidge 3-D chaotic system (1). Figs. 2-4 show the 2-D projection of the Rucklidge chaotic system (1) on the (x_1, x_2), (x_2, x_3), and (x_1, x_3) planes, respectively.

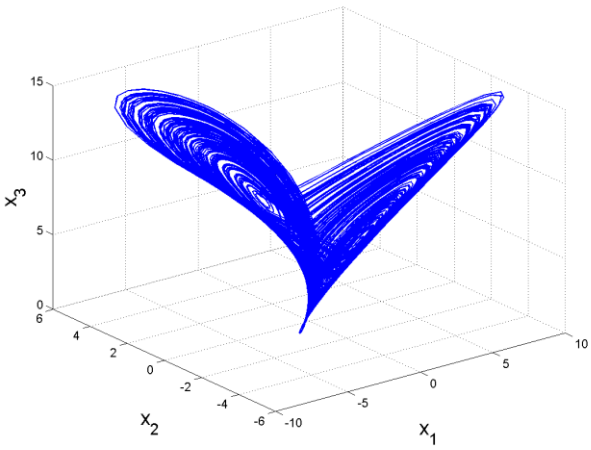

Fig. 1. Phase portrait of the Rucklidge 3-D chaotic system.

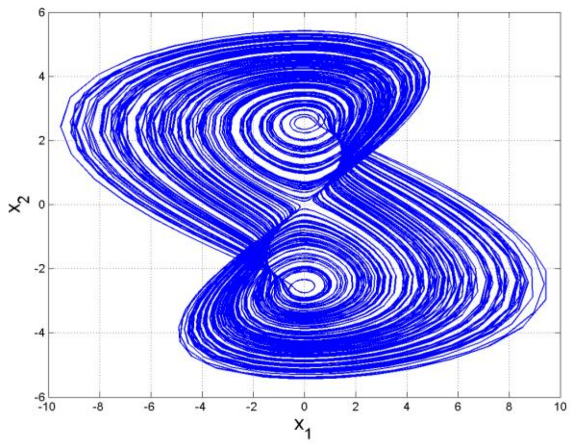

Fig. 2. 2-D projection of the Rucklidge chaotic system on the (x_1, x_2) plane.

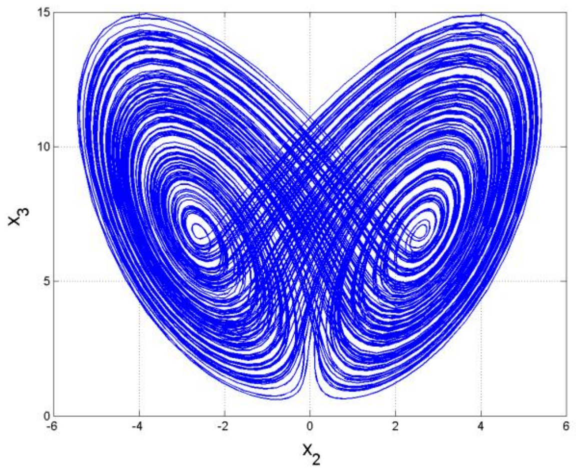

Fig. 3. 2-D projection of the Rucklidge chaotic system on the (x_2, x_3) plane.

3. Properties of the Rucklidge chaotic system

In this section, we discuss the qualitative properties of the Rucklidge chaotic system (1). We suppose that the parameter values of the Rucklidge system (1) are as in the chaotic case (2), *i.e.* $a = 2$ and $b = 6.7$.

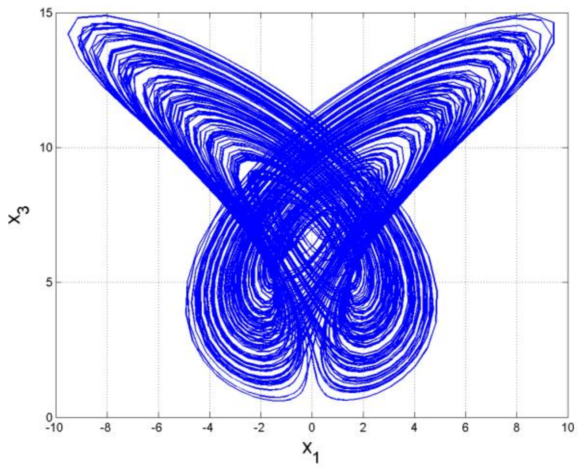

Fig. 4. 2-D projection of the Rucklidge chaotic system on the (x_1, x_3) plane.

3.1. Dissipativity

In vector notation, we may express the system (1) as

$$\dot{x} = f(x) = \begin{bmatrix} f_1(x_1, x_2, x_3) \\ f_2(x_1, x_2, x_3) \\ f_3(x_1, x_2, x_3) \end{bmatrix} \tag{6}$$

where

$$\begin{cases} f_1(x_1, x_2, x_3) = -ax_1 + bx_2 - x_2x_3 \\ f_2(x_1, x_2, x_3) = x_1 \\ f_3(x_1, x_2, x_3) = -x_3 + x_2^2 \end{cases} \tag{7}$$

Let Ω be any region in R^3 with a smooth boundary and also $\Omega(t) = \Phi_t(\Omega)$, where Φ_t is the flow of the vector field f. Furthermore, let $V(t)$denote the volume of $\Omega(t)$. By Liouville's theorem, we have

$$\dot{V} = \int_{\Omega(t)} (\nabla \cdot f)\, dx_1 dx_2 dx_3 \tag{8}$$

The divergence of the novel chaotic system (1) is easily found as

$$\nabla \cdot f = \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} + \frac{\partial f_3}{\partial x_3} = -a - 1 = -\mu \tag{9}$$

where

$\mu = a + 1 = 3 > 0.$

Substituting (9) into (8), we obtain the first order ODE

$$\dot{V} = -\mu V \tag{10}$$

Integrating (10), we obtain the unique solution as

$V(t) = \exp(-\mu t)V(0)$

for all $t \geq 0$ (11)

Since $\mu > 0$,it follows that $V(t) \to 0$ exponentially as $t \to \infty$. This shows that the 3-D novel chaotic system (1) is dissipative. Thus, the system limit sets are ultimately confined into a specific limit set of zero volume, and the asymptotic motion of the novel chaotic system (1) settles onto a strange attractor of the system.

3.2. Symmetry

It is easy to see that the system (1) is invariant under the coordinates transformation

$$(x_1, x_2, x_3) = (-x_1, -x_2, -x_3) \tag{12}$$

Thus, the system (1) exhibits *point reflection symmetry* about the origin in R^3.

3.3. Equilibrium points

The equilibrium points of the system (1) are obtained by solving the system of equations

$$\begin{cases} -ax_1 + bx_2 - x_2x_3 = 0 \\ x_1 = 0 \\ -x_3 + x_2^2 = 0 \end{cases} \tag{13}$$

Solving the system (13) with the values of the parameters as given in (2), we obtain three equilibrium points

$$E_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, E_1 = \begin{bmatrix} 0 \\ 2.5884 \\ 6.7000 \end{bmatrix}, E_2 = \begin{bmatrix} 0 \\ -2.5884 \\ 6.7000 \end{bmatrix} \tag{14}$$

The Jacobian of the system (1) at any point $x \in R^3$is given by

$$J(x) = \begin{bmatrix} -a & b - x_3 & -x_2 \\ 1 & 0 & 0 \\ 0 & 2x_2 & -1 \end{bmatrix} = \begin{bmatrix} -2 & 6.7 - x_3 & -x_2 \\ 1 & 0 & 0 \\ 0 & 2x_2 & -1 \end{bmatrix} \tag{15}$$

we find that

$$J_0 = J(E_0) = \begin{bmatrix} -2 & 6.7 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{16}$$

The eigenvalues of J_0 are numerically determined using MATLAB as

$$\lambda_1 = -1, \lambda_2 = -3.7749, \lambda_3 = 1.7749 \tag{17}$$

This shows that the equilibrium E_0 is a saddle-point, which is unstable. Next, we find that

$$J_1 = J(E_1) = \begin{bmatrix} -2 & 0 & -2.5884 \\ 1 & 0 & 0 \\ 0 & 5.1768 & -1 \end{bmatrix} \tag{18}$$

The eigenvalues of J_1 are numerically determined using MATLAB as

$$\lambda_1 = -3.5154, \lambda_{2,3} = 0.2577 \pm 1.9353i \tag{19}$$

This shows that the equilibrium point E_1 is a saddle-focus, which is unstable. We also find that

$$J_2 = J(E_2) = \begin{bmatrix} -2 & 0 & 2.5884 \\ 1 & 0 & 0 \\ 0 & -5.1768 & -1 \end{bmatrix} \tag{20}$$

The eigenvalues of J_2 are numerically determined as

$$\lambda_1 = -3.5154, \lambda_{2,3} = 0.2577 \pm 1.9353i \tag{21}$$

This shows that the equilibrium point E_2 is a saddle-focus, which is unstable.

3.4. Lyapunov exponents and kaplan-yorke dimension

We take the parameter values of the Rucklidge system (1) as in the chaotic case (2), i.e., $a = 2$ and $b = 6.7$.

We choose the initial values of the state as $x_1(0) = 1.2, x_2(0) = 0.8$and $x_3(0) = 1.4$.

Then we obtain the Lyapunov exponents of the system (1) using MATLAB as

$$L_1 = 0.1877, L_2 = 0, L_3 = -3.1893. \tag{22}$$

Fig. 5 shows the Lyapunov exponents of the system (1) as determined by MATLAB. We note that the sum of the Lyapunov exponents of the system (1) is negative. This shows that the Rucklidge chaotic system (1) is dissipative.

Also, the Maximal Lyapunov Exponent of the system (1) is $L_1 = 0.1877$.

The Kaplan-Yorke dimension of the Rucklidge chaotic system (1) is derived as

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.0589. \tag{23}$$

which is fractional.

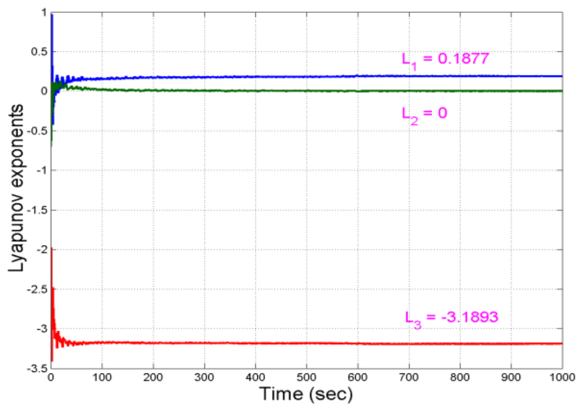

Fig. 5. Lyapunov exponents of the Rucklidge chaotic system.

4. Adaptive control design for the global stabilization of the Rucklidge time delayed chaotic system

In this section, we use adaptive control method to derive an adaptive feedback control law for globally and exponentially stabilizing the Rucklidge time delayed chaotic system with unknown parameters. Thus, we consider the Rucklidge time delayed chaotic system with controls given by

$$\begin{cases} \dot{x}_1 = -ax_1 + bx_2 - x_2x_3 + u_1 \\ \dot{x}_2 = x_1 + u_2 \\ \dot{x}_3 = -x_3(t-\tau) + x_2^2 + u_3 \end{cases} \quad 0 \le \tau \le 1 \tag{24}$$

In (24), x_1, x_2, x_3 are the states and u_1, u_2, u_3 are adaptive controls to be determined using estimates $\hat{a}(t)$, and $\hat{b}(t)$ for the unknown parameters $a$and b, respectively.

We consider the adaptive control law defined by

$$\begin{cases} u_1(t) = \hat{a}(t)x_1 - \hat{b}(t)x_2 + x_2x_3 - k_1x_1 \\ u_2(t) = -x_1 - k_2x_2 \\ u_3(t) = x_3(t-\tau) - x_2^2 - k_3x_3 \end{cases} \tag{25}$$

where k_1, k_2, k_3 are positive gain constants.

Substituting (25) into (24), we get the closed-loop plant dynamics as

$$\begin{cases} \dot{x}_1 = -[a - \hat{a}(t)]x_1 + [b - \hat{b}(t)]x_2 - k_1x_1 \\ \dot{x}_2 = -x_1 - k_2x_2 \\ \dot{x}_3 = x_3(t-\tau) - x_2^2 - k_3x_3 \end{cases} \tag{26}$$

The parameter estimation errors are defined as

$$\begin{cases} e_a(t) = a - \hat{a}(t) \\ e_b(t) = b - \hat{b}(t) \end{cases} \tag{27}$$

Using (27), we can simplify the plant dynamics (26) as

$$\begin{cases} \dot{x}_1 = -e_ax_1 + e_bx_2 - k_1x_1 \\ \dot{x}_2 = -x_1 - k_2x_2 \\ \dot{x}_3 = x_3(t-\tau) - x_2^2 - k_3x_3 \end{cases} \tag{28}$$

Differentiating (27) with respect to t,we obtain

$$\begin{cases} \dot{e}_a(t) = -\dot{\hat{a}}(t) \\ \dot{e}_b(t) = -\dot{\hat{b}}(t) \end{cases} \tag{29}$$

We use adaptive control theory to find an update law for the parameter estimates. We consider the quadratic candidate Lyapunov function defined by

$$V(\boldsymbol{x}, e_a, e_b) = \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2 + e_a^2 + e_b^2\right) \tag{30}$$

Clearly, $V$is a positive definite function on R^5.

Differentiating Valong the trajectories of (28) and (29), we obtain

$$\dot{V} = -k_1x_1^2 - k_2x_2^2 - k_3x_3^2 + e_a\left[-x_1^2 - \dot{\hat{a}}\right] + e_b\left[x_1x_2 - \dot{\hat{b}}\right] \tag{31}$$

In view of (31), we take the parameter update law as follows:

$$\begin{cases} \dot{\hat{a}} = -x_1^2 \\ \dot{\hat{b}} = x_1x_2 \end{cases} \tag{32}$$

Theorem 1: The Rucklidge 3-D chaotic system (24) with unknown system parameters is globally and exponentially stabilized for all initial conditions $\boldsymbol{x}(0) \in R^3$by the adaptive control law (25) and the parameter update law (32), where $k_1, k_2, k_3$are positive gain constants.

Proof: We prove this result by using Lyapunov stability theory (Khalil and Grizzle, 2002).

We consider the quadratic Lyapunov function defined by (30), which is positive definite on R^5. By substituting the parameter update law (32) into (31), we obtain the time derivative of Vas

$$\dot{V} = -k_1x_1^2 - k_2x_2^2 - k_3x_3^2 \tag{33}$$

From (33), it is clear that $\dot{V}$is a negative semi-definite function on R^5. Thus, we conclude that the state vector $\boldsymbol{x}(t)$and the parameter estimation error are globally bounded, *i.e.*

$$[\boldsymbol{x}(t) \quad e_a(t) \quad e_b(t)]^T \in L_\infty$$

We define $k = \min\{k_1, k_2, k_3\}$. Thus, it follows from (33) that

$$\dot{V} \le -k\|\boldsymbol{x}(t)\|^2 \tag{34}$$

Thus, we have

$$k\|\boldsymbol{x}(t)\|^2 \le -\dot{V} \tag{35}$$

Integrating the inequality (35) from 0to t,we get

$$k\int_0^t \|\boldsymbol{x}(\tau)\|^2 \, d\tau \le V(0) - V(t) \tag{36}$$

From (36), it follows that $\boldsymbol{x} \in \boldsymbol{L}_2$. Using (28), we can conclude that $\dot{\boldsymbol{x}} \in \boldsymbol{L}_\infty$. Using Barbalat's lemma (130), we can conclude that $\boldsymbol{x}(t) \to 0$exponentially as $t \to \infty$for all initial conditions $\boldsymbol{x}(0) \in R^3$. This completes the proof.

For numerical simulations, the classical fourth-order Runge-Kutta method with step size $h = 10^{-8}$ is used to solve the systems (24) and (32), when the adaptive control law (25) is applied. The parameter values of the novel chaotic system (24) are taken as in the chaotic case (2), i.e.

$$a = 2, b = 6.7 \tag{37}$$

We take the positive gain constants as $k_i = 5$for $i = 1,2,3$.

Furthermore, as initial conditions of the Rucklidge chaotic system (24), we take

$$x_1(0) = 12.3, x_2(0) = -22.7, x_3(0) = 16.4 \tag{38}$$

Also, as initial conditions of the parameter estimates, we take

$$\hat{a}(0) = 15.8, \hat{b}(0) = 24.9 \tag{39}$$

Fig. 6 shows the exponential convergence of the controlled state trajectories of the Rucklidge chaotic system (24).

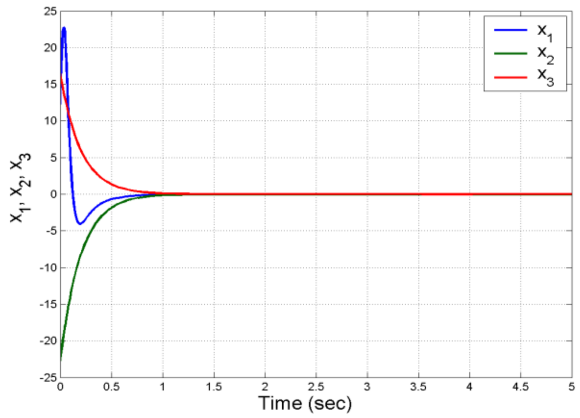

Fig. 6. Time-history of the controlled state trajectories of the novel chaotic system.

5. Adaptive synchronization of the Rucklidge chaotic system with Rucklidge time delayed chaotic system

In this section, we use adaptive control method to derive an adaptive feedback control law for globally synchronizing identical 3-D Rucklidge chaotic systems with unknown parameters. As the master system, we consider the Rucklidge chaotic system given by

$$\begin{cases} \dot{x}_1 = -ax_1 + bx_2 - x_2x_3 \\ \dot{x}_2 = x_1 \\ \dot{x}_3 = -x_3 + x_2^2 \end{cases} \tag{40}$$

where $x_1, x_2, x_3$are the states and $a, b$are unknown system parameters.

As the slave system, we consider the controlled Rucklidge time delayed chaotic system given by

$$\begin{cases} \dot{y}_1 = -ay_1 + by_2 - y_2y_3 + u_1 \\ \dot{y}_2 = y_1 + u_2 \\ \dot{y}_3 = -y_3(t-\tau) + y_2^2 + u_3 \end{cases} \tag{41}$$

where y_1, y_2, y_3 are the states and u_1, u_2, u_3 are adaptive controls to be determined using estimates $\hat{a}(t)$, $\hat{b}(t)$ for the unknown system parameters a, b, respectively.

The synchronization error between the novel chaotic systems (40) and (41) is defined by

$$\begin{cases} e_1 = y_1 - x_1 \\ e_2 = y_2 - x_2 \\ e_3 = y_3 - x_3 \end{cases} \tag{42}$$

Then the error dynamics is obtained as

$$\begin{cases} \dot{e}_1 = -ae_1 + be_2 - y_2y_3(t-\tau) + x_2x_3 + u_1 \\ \dot{e}_2 = e_1 + u_2 \\ \dot{e}_3 = -e_3 + y_2^2 - x_2^2 + u_3 \end{cases} \tag{43}$$

We consider the adaptive feedback control law

$$\begin{cases} u_1 = \hat{a}(t)e_1 - \hat{b}(t)e_2 + y_2y_3(t-\tau) - x_2x_3 - k_1e_1 \\ u_2 = -e_1 - k_2e_2 \\ u_3 = e_3 - y_2^2 + x_2^2 - k_3e_3 \end{cases} \tag{44}$$

where k_1, k_2, k_3 are positive constants and $\hat{a}(t)$, $\hat{b}(t)$ are estimates of the unknown parameters a, b, respectively.

Substituting (44) into (43), we can simplify the error dynamics (43) as

$$\begin{cases} \dot{e}_1 = -[a - \hat{a}(t)]e_1 + [b - \hat{b}(t)]e_2 - k_1e_1 \\ \dot{e}_2 = -k_2e_2 \\ \dot{e}_3 = -k_3e_3 \end{cases} \tag{45}$$

The parameter estimation errors are defined as

$$\begin{cases} e_a = a - \hat{a}(t) \\ e_b = b - \hat{b}(t) \end{cases} \tag{46}$$

Substituting (46) into (45), the error dynamics is simplified as

$$\begin{cases} \dot{e}_1 = -e_ae_1 + e_be_2 - k_1e_1 \\ \dot{e}_2 = -k_2e_2 \\ \dot{e}_3 = -k_3e_3 \end{cases} \tag{47}$$

Differentiating (43) with respect to t,we obtain

$$\begin{cases} \dot{e}_a = -\dot{\hat{a}}(t) \\ \dot{e}_b = -\dot{\hat{b}}(t) \end{cases} \quad (48)$$

We consider the quadratic candidate Lyapunov function defined by

$$V(\boldsymbol{e}, e_a, e_b) = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_a^2 + e_b^2\right) \quad (49)$$

Differentiating V along the trajectories of (47) and (48), we obtain

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 + e_a\left[-e_1^2 - \dot{\hat{a}}\right] + e_b\left[e_1 e_2 - \dot{\hat{b}}\right] \quad (50)$$

In view of (50), we take the parameter update law as follows.

$$\begin{cases} \dot{\hat{a}} = -e_1^2 \\ \dot{\hat{b}} = e_1 e_2 \end{cases} \quad (51)$$

Next, we state and prove the main result of this section.

Theorem 2: The Rucklidge chaotic systems (40) and (41) with unknown system parameters are globally and exponentially synchronized for all initial conditions $\boldsymbol{x}(0), \boldsymbol{y}(0) \in R^3$ by the adaptive control law (44) and the parameter update law (51), where k_1, k_2, k_3 are positive constants.

Proof: We prove this result by applying Lyapunov stability theory (Khalil and Grizzle, 2002).

We consider the quadratic Lyapunov function defined by (49), which is positive definite on R^5. By substituting the parameter update law (51) into (50), we obtain the time-derivative of V as

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 \quad (52)$$

From (52), it is clear that $\dot{V}$ is a negative semi-definite function on R^5. Thus, we can conclude that the synchronization error vector $\boldsymbol{e}(t)$ and the parameter estimation error are globally bounded, *i.e.*

$$\left[\boldsymbol{e}(t) \quad e_a(t) \quad e_b(t)\right]^T \in L_\infty \quad (53)$$

We define $k = \min\{k_1, k_2, k_3\}$. Then it follows from (52) that

$$\dot{V} \le -k\|\boldsymbol{e}(t)\|^2 \quad (54)$$

Thus, we have

$$k\|\boldsymbol{e}(t)\|^2 \le -\dot{V} \quad (55)$$

Integrating the inequality (55) from 0 to t, we get

$$\int_0^t k\|\boldsymbol{e}(\tau)\|^2 d\tau \le V(0) - V(t) \quad (56)$$

From (56), it follows that $\boldsymbol{e} \in L_2$. Using (47), we can conclude that $\dot{\boldsymbol{e}} \in L_\infty$.

Using Barbalat's lemma (Khalil and Grizzle, 2002), we conclude that $\boldsymbol{e}(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $\boldsymbol{e}(0) \in R^3$. This completes the proof.

For numerical simulations, we take the parameter values of the chaotic systems (40) and (41) as in the chaotic case (2), *i.e.* $a = 2$ and $b = 6.7$.

We take the positive gain constants as $k_i = 5$ for $i = 1,2,3$. As initial conditions of the master system (40), we take

$$x_1(0) = 12.4, x_2(0) = -3.1, x_3(0) = -17.8 \quad (57)$$

As initial conditions of the slave system (41), we take

$$y_1(0) = 5.7, y_2(0) = 19.2, y_3(0) = 26.5 \quad (58)$$

As initial conditions of the parameter estimates, we take

$$\hat{a}(0) = 15.4, \hat{b}(0) = 17.8 \quad (59)$$

Figs. 7-9 depict the synchronization of the Rucklidge chaotic systems (40) and (41). Fig. 10 depicts the time-history of the complete synchronization errors e_1, e_2, e_3.

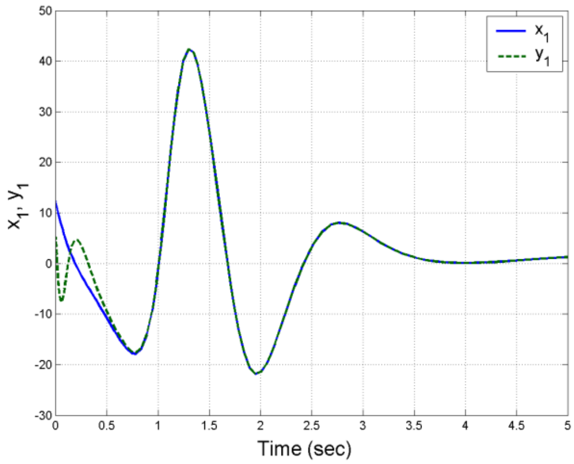

Fig. 7. Synchronization of the states x_1 and y_1.

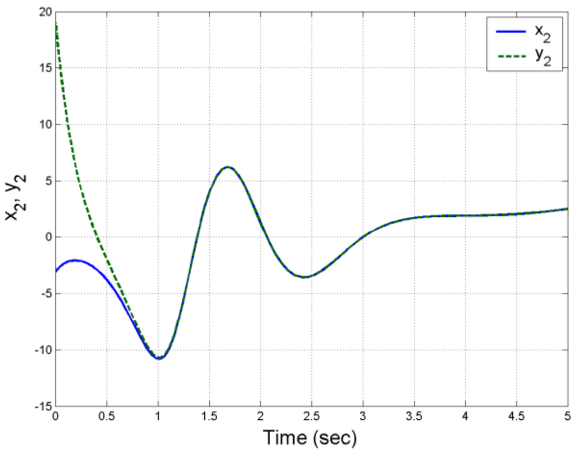

Fig. 8. Synchronization of the states x_2 and y_2.

6. Circuit simulation and LabView implementation

6.1. LabView implementation of adaptive control design for the global stabilization of the Rucklidge time delayed chaotic system

In this section, the adaptive control method to derive an adaptive feedback control law for globally and exponentially stabilizing the Rucklidge time-delayed chaotic system with unknown parameters was discussed in section 4. Fig. 11 shows the Rucklidge time delayed system implemented in LabVIEW using the Control Design and Simulation Loop. Fig. 12 the designed controller for stabilizing the time delayed chaotic system is implemented in LabVIEW using the Control Design and

Simulation loop. Fig. 13 shows the 2D Phase portraits X3X1, X1X2, X2X3 of the Rucklidge time delayed System. Fig. 14 shows the stabilized states of the Rucklidge time delayed system.

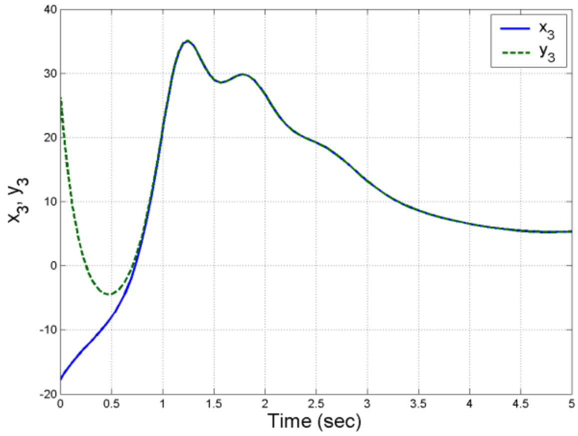

Fig. 9. Synchronization of the states $x_3$and y_3.

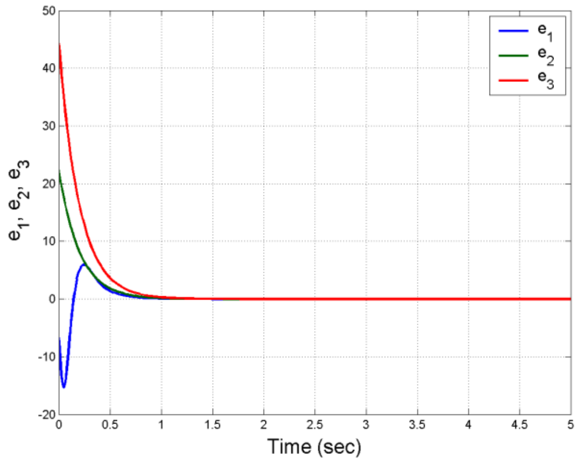

Fig. 10. Time-history of the synchronization errorse$_1$, e_2, e_3.

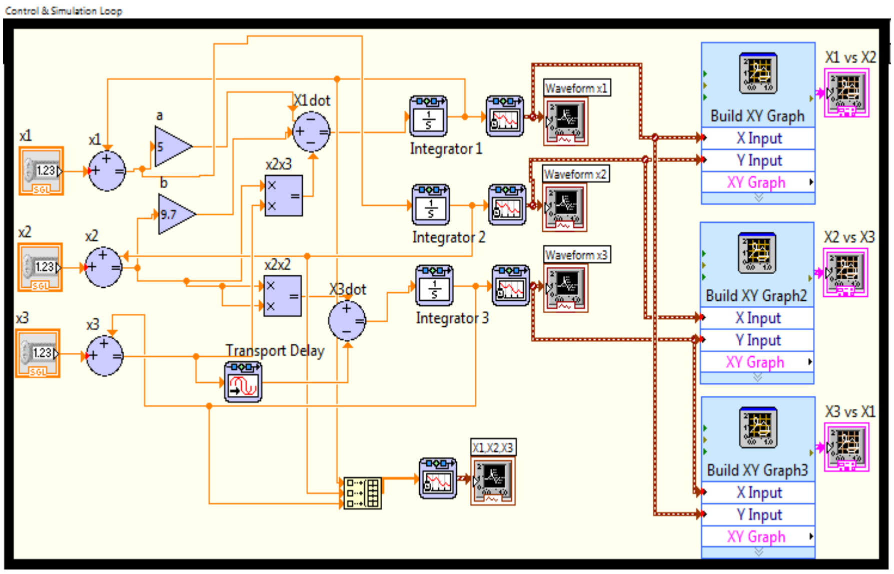

Fig. 11. Block diagram of the Rucklidge time delayed system.

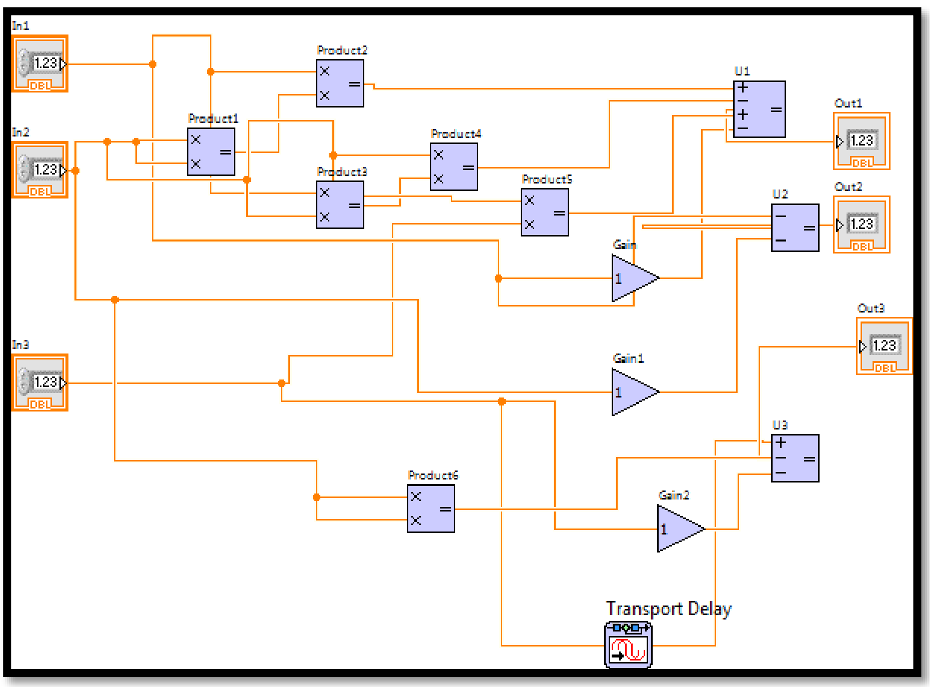

Fig. 12. Block diagram of the controller as subsystem.

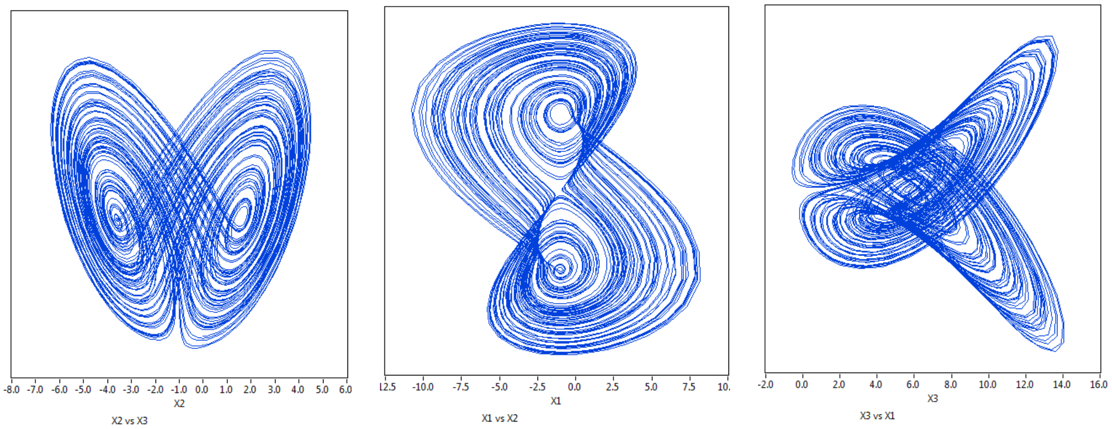

Fig. 13. 2D phase portraits X3X1, X1X2, X2X3 of the Rucklidge system.

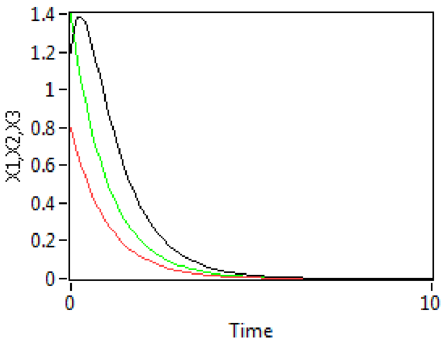

Fig. 14. Stabilized state phase of the Rucklidge system.

7. LabView implementation of adaptive synchronization of the Rucklidge chaotic systems with Rucklidge time delayed chaotic systems

In this section, the adaptive control method to derive an adaptive feedback control law for globally synchronizing 3-D Rucklidge chaotic systems with Rucklidge time delayed chaotic systems discussed in section 5 is Implemented using LabVIEW. Fig. 15 shows the Slave subsystem. Fig. 16 shows the Designed adaptive controller U. Fig. 17 shows the time history of the synchronization errors.

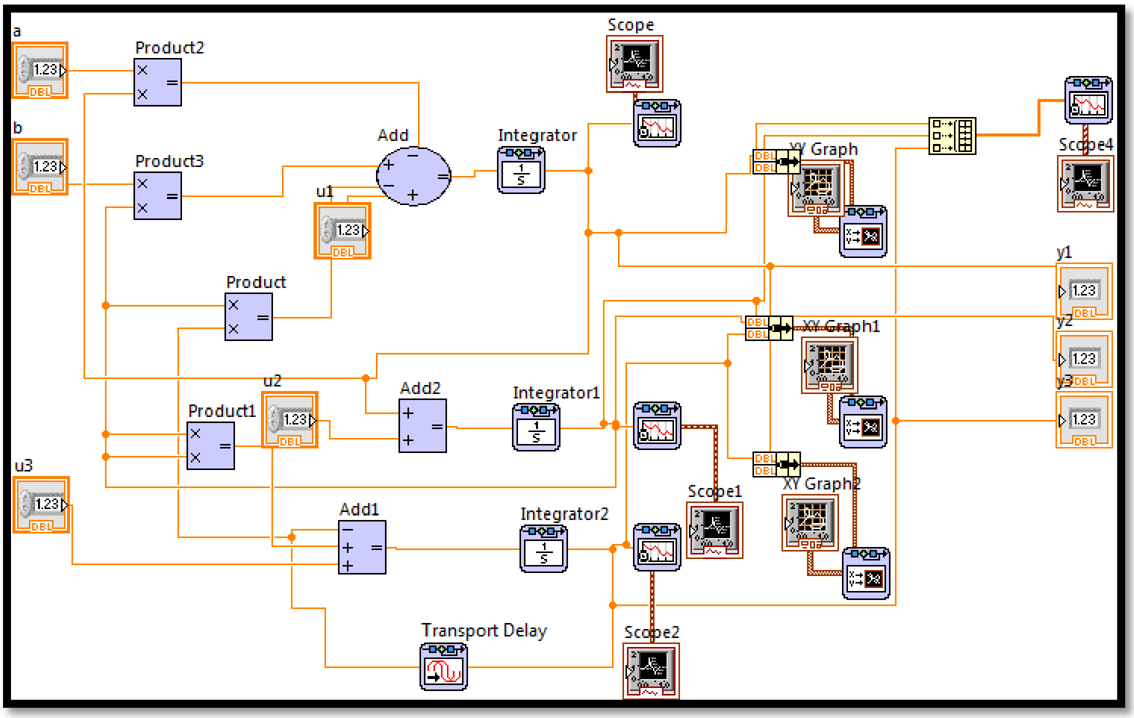

Fig. 15. Block diagram of the slave Rucklidge time delayed system.

8. Conclusion

In this paper, a detailed description of the Rucklidge's nonlinear double convection system and the parameter values for which the Rucklidge system exhibits chaotic behavior is discussed. Next, an adaptive feedback controller is designed for the global chaos control of the Rucklidge time delayed chaotic system with unknown parameters. Furthermore, an adaptive feedback controller is designed for the global chaos synchronization of the identical Rucklidge chaotic system with Rucklidge time delayed chaotic system. All the main results derived in this work are illustrated with MATLAB simulations. Finally, a circuit design of the novel 3-D chaotic system is implemented in LABVIEW to validate the theoretical chaotic model.

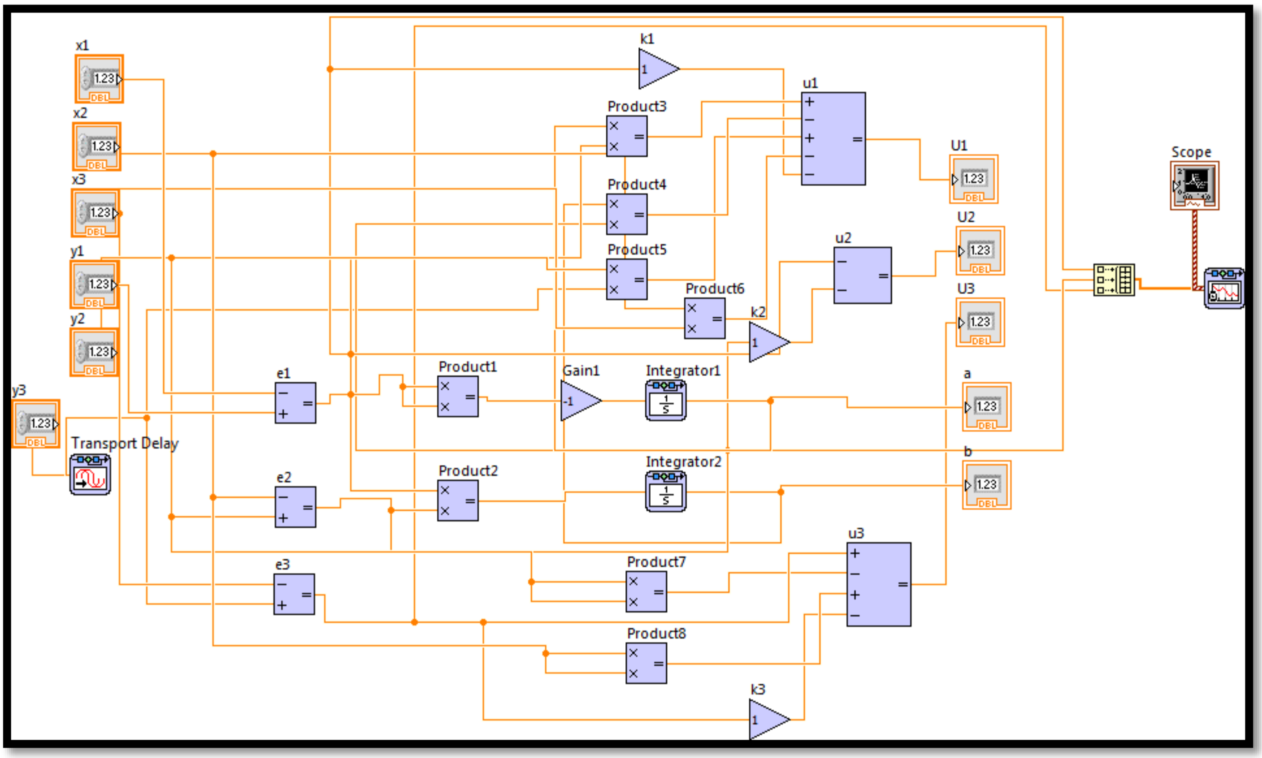

Fig. 16. Block diagram of the synchronization controller.

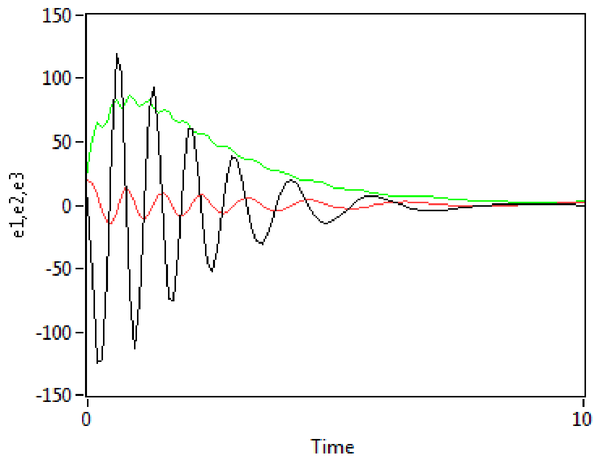

Fig. 17. Time history of the Synchronization errors.

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